

## Creating novel dynamic architectural forms from kinetic elastica-ruled surfaces

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### Abstract

Dynamic architectural forms are studied for many novel applications, as they can reconfigure their shapes to generate striking visual effects and obtain improved structural performance to respond to environmental changes. Such architectural forms may be created using the elastic-kinetic approach to achieve complex elastic deformations without mechanically complex systems. In this paper, we propose a new dynamic architectural form based on the concept of "kinetic elastica-ruled surface," which can rationalize dynamic curved surfaces using a series of non-interacting elastic strips bent to their minimum energy states. We classify the formed surfaces into two categories based on their motion behavior: "distanced-based" and "rotation-based" kinetic elastica-ruled surfaces. We demonstrate that distanced-based kinetic elastica-ruled surfaces can achieve interesting wave effects simply by controlling the support distances of parallel elastic strips. Moreover, we show that rotation-based kinetic elastica-ruled surfaces can alter their shapes by controlling the radial motion of the boundaries using non-parallel elastic strips. A full-scale rotation-based kinetic elastica-ruled surface is built to demonstrate its capability to realize a lightweight, high-speed, and cost-effective construction.

**Keywords:** dynamic architectural form, parametric modeling, elastica, curved ruling, complex surface

### 1. Introduction

Ruled surfaces create elegant 3D forms by simply sweeping straight lines along the path curves (directrices) [1]. This surface generation process has been widely adopted to create numerous innovative and aesthetically pleasing architectural forms [2]. Complex 3D surfaces created by straight rulings can be rationalized by a set of line segments, thereby offering a cost-effective solution to convert smooth surfaces to producible elements [3]. Straight rulings also accord with robotic hot-wire cutting techniques [4]. Such methods can offer efficient solutions to realize ruled surfaces by using straight wire cutting.

Sweeping curved lines along the directrices can also create novel architectural forms [5, 6]. A new type of ruled surface, termed elastica-ruled surface (see Figure 3), has been recently proposed [7]. By replacing straight rulings with elastically deformed curves, line-ruled surfaces are transformed into elastica-ruled surfaces. The shape of each elastica curve can be accurately determined based on specifying any two of the following design parameters: support distance  $b$ , arc length  $L$ , curve height  $h$ , and the initial tangent angle  $\Theta$ . The orientation of the elastica curve around the straight ruling is controlled by another independent parameter  $\beta$ . Compared to the straight rulings, the set of design options created by curved rulings can be largely increased by altering the elastica curvature and orientation of the elastica curves.

It is worth pointing out that elastica curves are 2D shapes corresponding to elastic deformations bent from a straight slender beam. Due to this, using movable directrices in an elastica-ruled surface can continuously change the bent states of elastica rulings, leading to a new type of dynamic form, termed kinetic elastica-ruled surface [7]. Shapes of kinetic elastica-ruled surfaces during motion can be fully controlled by parameters  $b$ ,  $h$ ,  $\Theta$ , and/or  $\beta$ , with a fixed  $L$ . In doing so, the formed surface can be reconfigured to a shape that satisfies aesthetic or functional requirements. The kinetic feature also enables rapid on-demand response to environmental changes. Elastic materials, such as fiber-reinforced plastic, polycarbonate, and flexible metal plates, can be used in constructing such kinetic designs, resulting in lightweight, high-speed, and cost-effective constructions.

In this study, we propose a new dynamic architectural form based on the concept of kinetic elastica-ruled surfaces. The generated dynamic surfaces are classified into two categories, including "distanced-based" and "rotation-based"; both can lead to striking visual effects. To demonstrate potential practical applications of the proposed concept, we show a full-scale dynamic pavilion designed based on rotation-based kinetic elastica-ruled surfaces, as shown in Figure 1. The remainder of the paper is organized as follows: Section 2 describes the numerical definition of kinetic elastica-ruled surfaces. Section 3 presents the design and construction details of the proposed pavilion. The conclusions are drawn in Section 4 to highlight the important features of the proposed dynamic architectural form.



Figure 1: A full-scale dynamic pavilion designed based on rotation-based kinetic elastica-ruled surfaces.

## **2. Creation of kinetic elastica-ruled surfaces**

### **2.1. Static elastica-ruled surfaces**

Elastica curves describe the naturally stable forms of a bent elastic strip, whose configuration has minimum bending energy and non-uniform curvature (Figure 2a). Under different boundary conditions, elastica curves can be bent into various shapes. Such elastic deformations can be used to create novel architectural forms for both static and kinetic applications. A unique feature of elastica curves is their material independency, meaning different materials can reach the same bent state using equivalent structural systems.

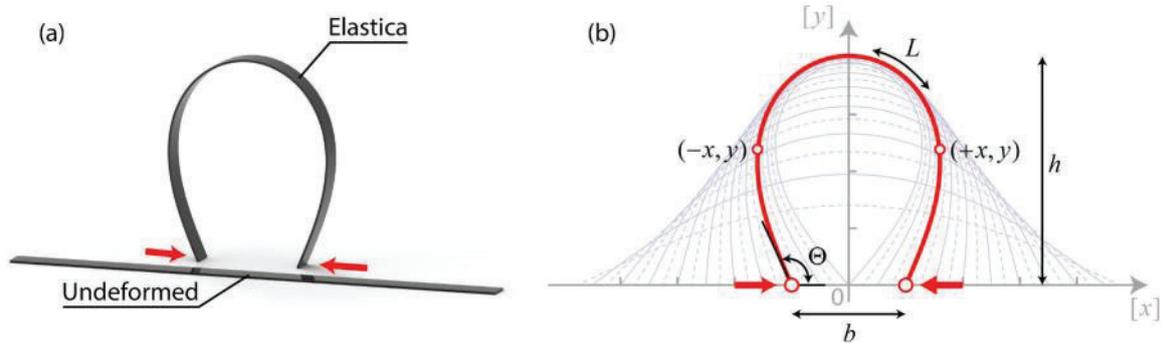


Figure 2: Elastica curves: (a) a 3D beam before and after bending; (b) design parameters for an elastica curve.

The analytical solutions of pinned-pinned elastica curves with a uniform bending stiffness are well studied in the literature [8, 9]. Their mathematical expression is derived based on Jacobi elliptic functions using elliptic integrals. As shown in Figure 2b, each bent state can be described by four curve design parameters described in Section 1. Based on the mathematical relationship, specifying any two curve design parameters gives the solutions for the other two unknowns. Hence, the designers can control the shape of an elastica curve using only two curve design parameters as variables. Once all curve design parameters are determined, they can be converted to  $(\pm x, y)$  coordinates in a cartesian system for describing the curve shape [7].

Elastica-ruled surfaces are generated by transforming line-ruled surfaces using curved rulings. As shown in Figure 3a, the first step of the transformation process requires a line-ruled surface to be predetermined using directrices and rulings. The length of a straight ruling represents the support distance  $b$ , and an additional curve design parameter is needed to define an elastica curve (see Figure 3b). The generated elastica curve can be rotated around the straight ruling by an angle  $\beta$ . Figure 3c shows the elastica curves generated on all straight rulings, forming a complete set of curved rulings. By sweeping the curved rulings, a continuous elastica-ruled surface is constructed (see Figure 3d). As a straight line is the elastica in a relaxed state, line-ruled surfaces can be regarded as a special case of elastica-ruled surfaces.

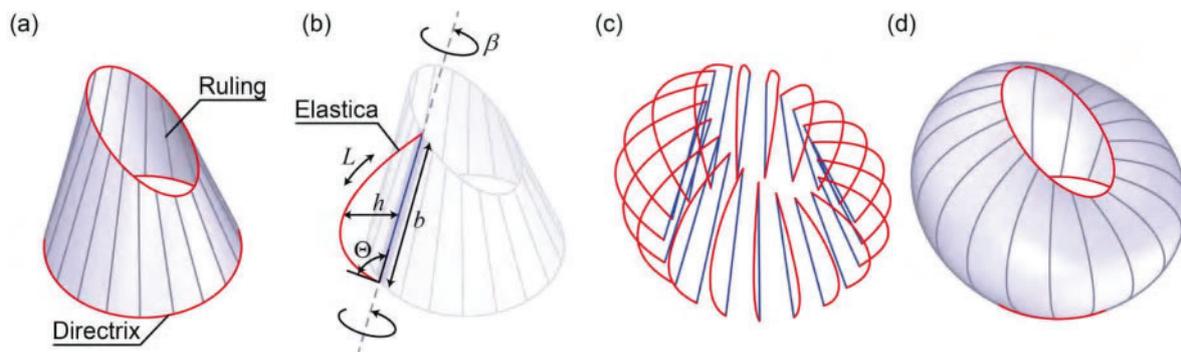


Figure 3: Generation of 3D elastica-ruled surfaces: (a) line-ruled surfaces; (b) generate an elastica curve on a straight ruling (c) curved rulings (red) and straight rulings (blue); (d) elastica-ruled surfaces.

## 2.2. Kinetic elastica-ruled surfaces

Kinetic elastica-ruled surfaces are created by dynamically changing the bent states of the curved rulings of elastica-ruled surfaces. Such structures can be reconfigured to different shapes to enhance their aesthetic qualities and respond to environmental changes.

This paper uses flexible strip elements as the structural skeleton, whose behavior can be predicted using the elastica theory. The length of each strip can be predetermined, making the arc length  $L$  a fixed parameter. Therefore, one of the other three parameters— $b$ ,  $h$ , or  $\Theta$ —must be varied continuously to create smooth dynamic effects. A mechanically viable solution to achieving such dynamic effects is to modify the relative position of the directrices, thereby changing the support distance  $b$  of all elastica rulings. Depending on the motion behavior of the directrices, kinetic elastica-ruled surfaces are classified into two categories: distance-based and rotation-based.

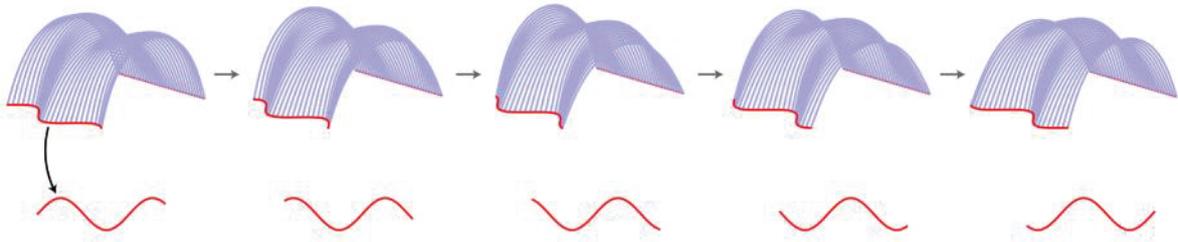
### *2.2.1. Distance-based kinetic elastica-ruled surfaces*

Distance-based kinetic elastica-ruled surfaces use parallel elastic strips to create dynamic effects by changing the shape of the directrices. Different bent states of elastica curves are achieved by adjusting the support distance  $b$  in the same plane. With the strip length  $L$  fixed and support distance  $b$  determined, we can calculate the values of  $h$  and  $\Theta$  to capture the overall profile of the formed surface. It should be noted that sudden shape changes in the directrices may result in large gaps between consecutive strips and should be avoided.

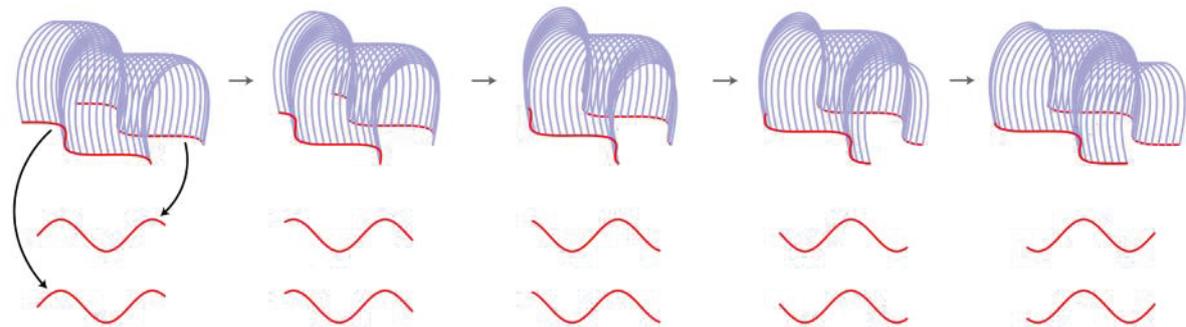
Mechanically, 1-DOF end actuators should be attached to the supports to control the opening distances of bent strips. Rollers can be installed between the end actuators and the strips to ensure pinned-pinned connections. The moving speed of end supports can also affect the dynamic visual effect but should be carefully controlled to prevent the structure from swaying.

Based on the proposed mechanical system, there are three ways to move the end supports: (i) one dynamic directrix, (ii) two dynamic directrices with the same movement, and (iii) two dynamic directrices with different movements, as shown in Figure 4a-c, respectively. Among these three types, type (i) may be less expensive, as it only needs to trigger half of the actuators; types (ii) and (iii) require all actuators to be active. In order to significantly reduce the construction costs, we may change the bent states of elastic strips using non-deformable directrices so that we only need a small number of actuators, as discussed further below.

(a) Changing only one side



(b) Changing simultaneously



(c) Changing separately

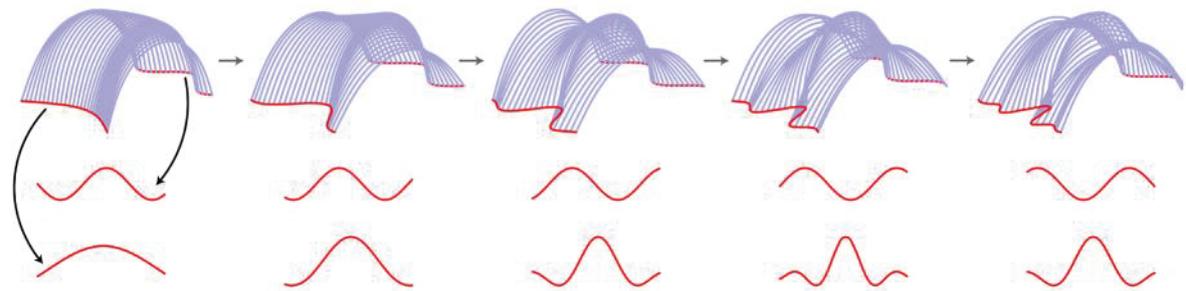


Figure 4: Distance-based kinetic elastica-ruled surfaces: (a) one dynamic directrix; (b) two dynamic directrices with the same movement; (c) two dynamic directrices with different movements.

### 2.2.2. Rotation-based kinetic elastica-ruled surfaces

Rotation-based kinetic elastica-ruled surfaces transform the shape of non-parallel elastic strips by rotating the non-deformable directrices around the given axes, meaning the directrices are considered rigid bodies. As shown in Figure 5, the continuous motion of the formed surface can be generated by rotating one directrix or two directrices, thereby changing the support distance of elastica curves.

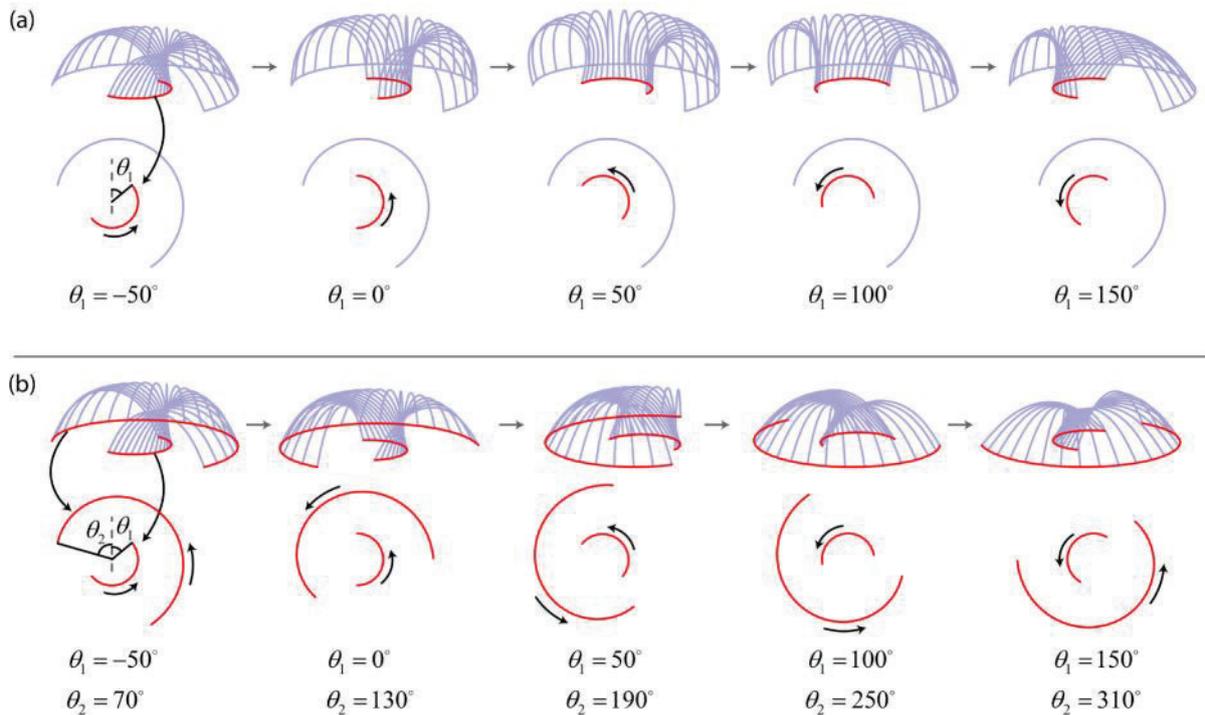


Figure 5: Rotation-based kinetic elastica-ruled surfaces: (a) the outer directrix is fixed, and the inner directrix is rotatable; (b) both directrices are rotatable.

Figure 5a shows a dynamic form with a rotatable inner directrix and a fixed outer directrix. At the initial position ( $\theta_1 = -50^\circ$ ), the structure has a lower overall height. After rotation ( $\theta_1 = 50^\circ$ ), the support distances of strips are reduced, resulting in higher curve heights. Further rotation movements broaden the entrance on the left side while lowering the height on the other side. If both directrices are allowed to be rotated, the dynamic form has much greater freedom of movement, as shown in Figure 5b. Besides, rotating the outer directrix can change the orientation of the entrances so that the dynamic form can respond to different environmental conditions.

Mechanically, the inner or outer directrix can be mounted onto a rail, and the rotation movements can be powered by motors. Each strip support should be carefully designed to allow free rotation so that pinned-pinned elastica can be achieved during motion. Otherwise, the rotation of the directrix can induce twisting near the supports and triggers unwanted elastic deformations of strips.

### 3. Application to a full-scale pavilion

#### 3.1. Design

To demonstrate the potential applications of kinetic elastica-ruled surfaces, we have constructed a full-scale pavilion using rotation-based elastica-ruled surfaces, as shown in Figure 6. A turntable representing the inner directrix is used to achieve the rotation movement of the pavilion. Note that we adjust the turntable manually to control the shape of the formed surface; we plan to introduce a motor system into this design in the future to realize an interactive architectural form.

The pavilion has twenty  $5,000 \text{ mm} \times 140 \text{ mm}$  elastic strips made using 10 mm thick polycarbonate sheets, which can achieve desired elastic deformations. Each strip can also be rolled into a cylinder form with a 1.5 m diameter for transportation. The supports of the strips are designed to be rotatable pinned connections, as shown in Figure 6.

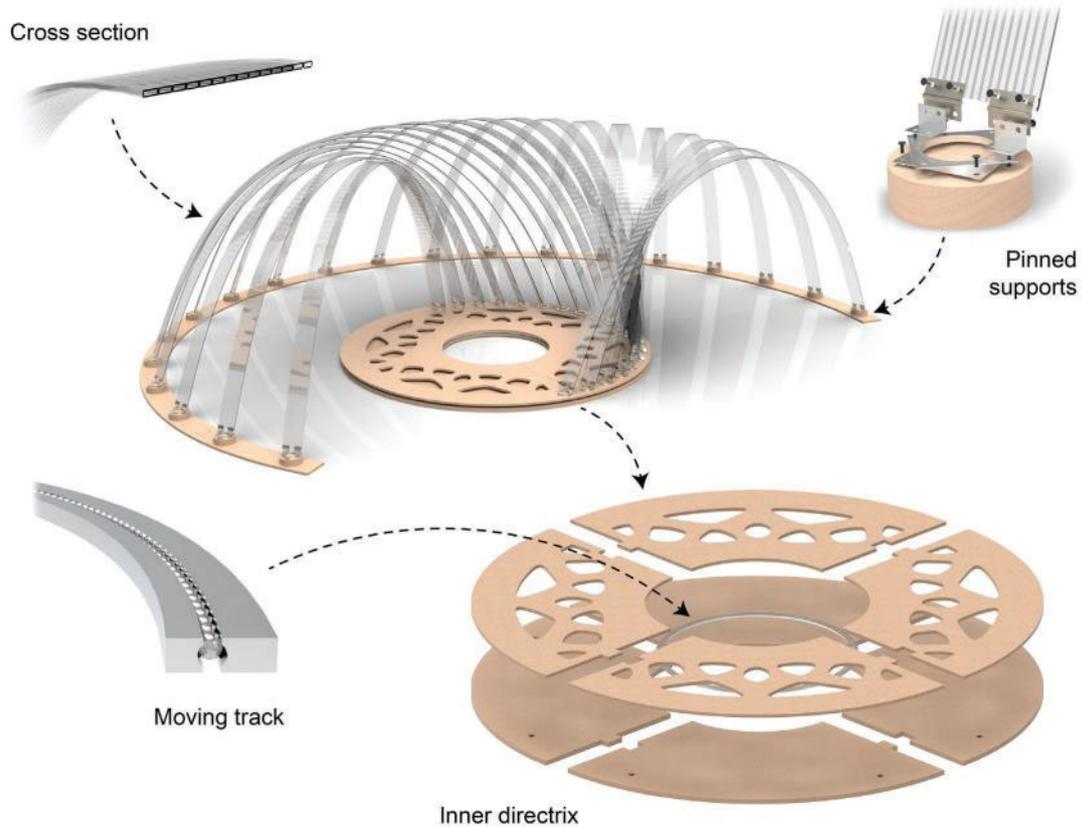


Figure 6: Components of the proposed pavilion designed based on a rotation-based kinetic elastica-ruled surface.

In order to fulfill the transportation requirement of the IASS design competition, the outer boundary (representing the outer directrix) is partitioned into ten segments and designed with an interlocking mechanism, as shown in Figure 7. Each segment has a dimension of approximately  $1.2 \text{ m} \times 0.2 \text{ m}$ . All segments are cut using a CNC machine and nested in a  $3 \text{ m} \times 1.3 \text{ m}$  rectangular region to minimize material waste. The outer boundary is cut from an 18 mm thick medium-density fibreboard (MDF). Small cylinders with the same material are installed on the outer boundary to match the height of the turntable. The assembled outer boundary is tested to be "heavy enough" so it can remain in its position while the turntable and bent strips are moved.

The turntable (representing the inner directrix) is made up of three layers: a circular MDF base (bottom), a metal moving track (middle), and an MDF loading platform (top), as shown in Figure 6. The base layer and the loading platform are cut in quarters for ease of transportation, and an interlocking mechanism is designed for their assembly, as shown in Figure 6. It is worth pointing out that the loading platform is designed using topology optimization with inefficient materials removed to reduce the overall weight and improve the aesthetic quality. Specifically, it is obtained using the bi-directional evolutionary structural optimization method [10] and is 40% lighter than a solid plate. The moving track contains metal balls with little friction for realizing smooth rotation movements. Together, the assembled turntable has a 2.6 m diameter and 60 mm thickness. It is heavy, so it cannot be moved while other parts are moving.

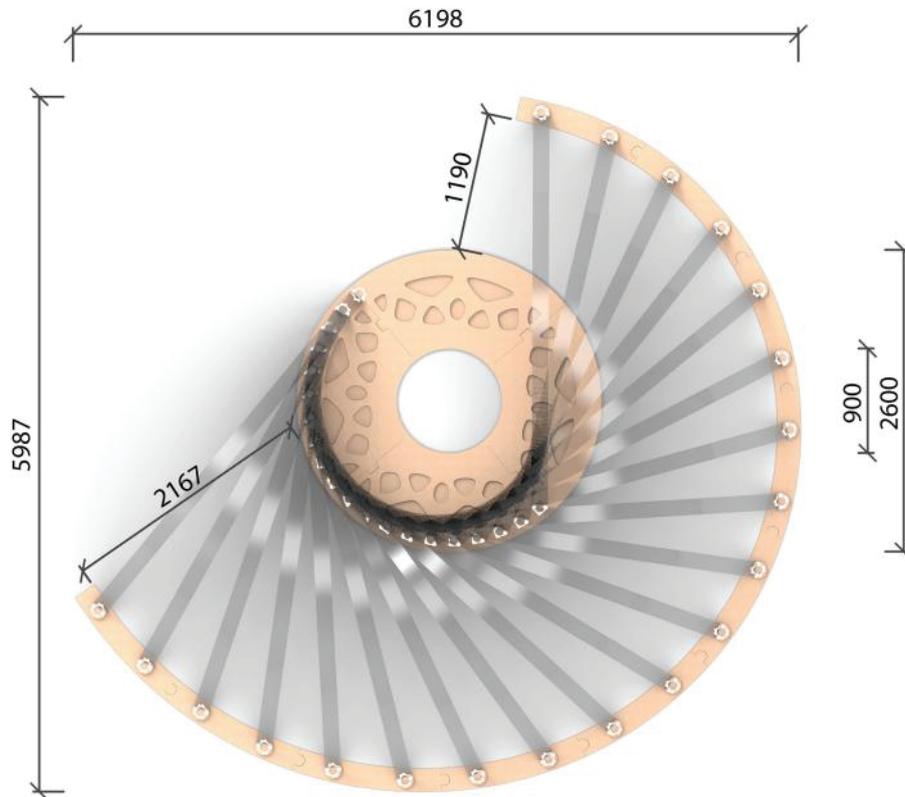


Figure 7: Top view of the pavilion design.

### 3.2. Construction details

All construction materials were purchased from local suppliers. After acquiring all the materials, the pavilion was assembled on the RMIT University campus. As illustrated in Figure 8, the construction involves the following steps:

- (1) We cut MDF components using a CNC machine, including the inner base, loading platform, and outer boundary. A 0.2 mm tolerance was specified to allow easy assembly and disassembly.
- (2) We installed rotatable connections at the polycarbonate strip ends. Each connection included several components, including one small turntable, two L-brackets, and two butt hinges.
- (3) We assembled both inner and outer MDF bases and placed them in the correct position.
- (4) We installed the moving track on the inner MDF and connected them using bolts.
- (5) We assembled the loading platform and connected it to the moving track using bolts.
- (6) We connected all 20 elastic strips to the inner and outer assembled boundaries using screws to complete the construction.

Note that, in order to change the shape of the pavilion, the turntable was manually rotated.



Figure 8: The construction details of the pavilion: (1) cut MDF boards using a CNC machine; (2) install rotatable connections at the strip ends; (3) place the MDF bases on site; (4) install the moving track; (5) assemble the turntable; (6) connect elastic strips to the inner and outer assembled boundaries.

### 3.3. Results and discussion

Figure 9 shows the pavilion with the inner directrix rotated at different angles. The error in camera position and construction has led to a slight mismatch between the simulated and constructed results. It can be seen that the elastic deformations of the strips match well with the simulations, confirming that complex dynamic forms can be simply and inexpensively achieved using the concept of the kinetic elastica-ruled surfaces. When viewed from outside the structure, the kinetic movement of the structure generates visually captivating effects. Inside the structure, users can rotate the turntable to change the "shape" of their surroundings. Interestingly, as the polycarbonate strips are transparent, walking inside the pavilion feels like being protected by a barrel wave. A more interesting visual effect is with LED lights on, the pavilion became an array of evolving light strips at night (see Figure 1).

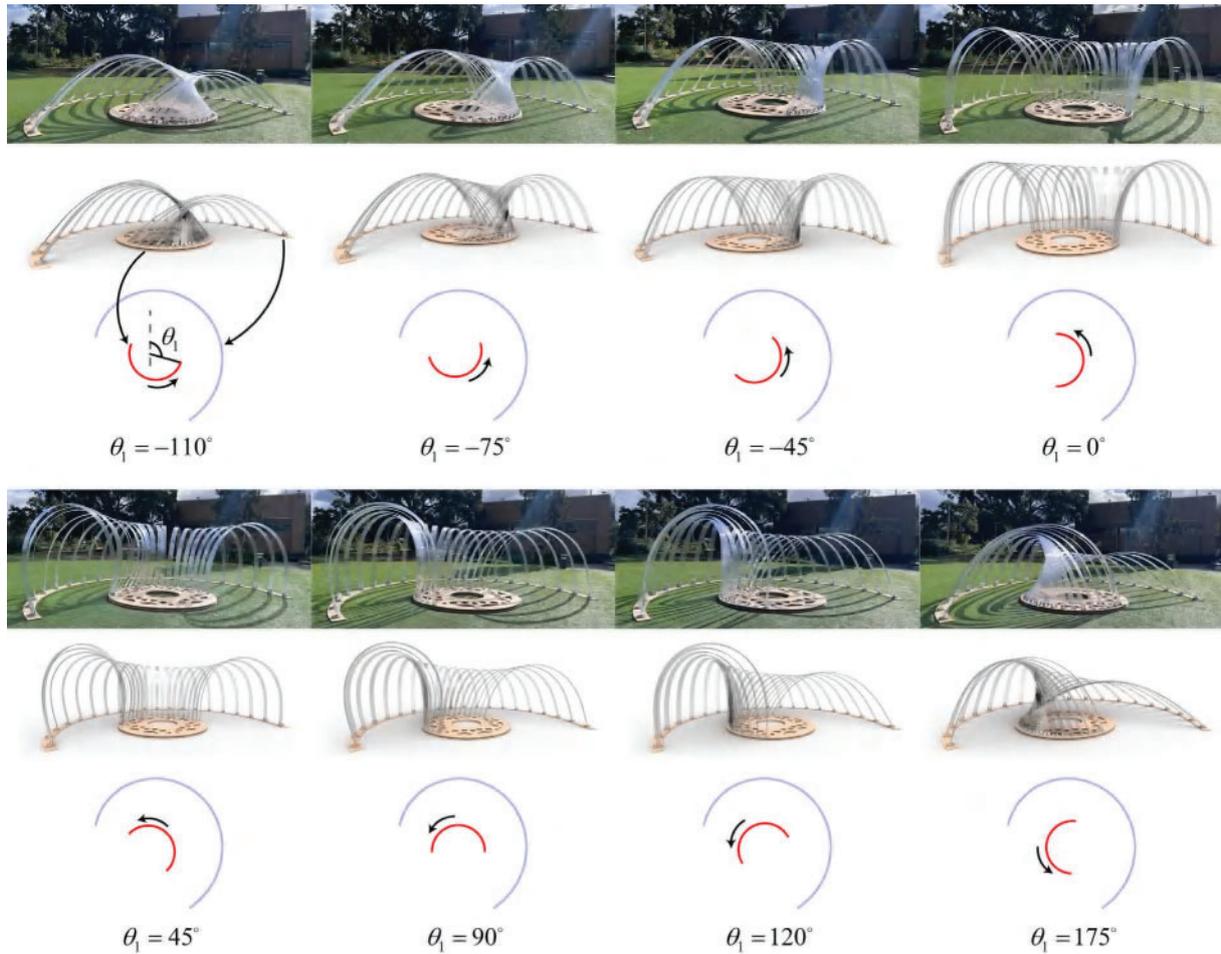


Figure 9: The dynamic pavilion with the inner directrix rotated at different angles.

### 3.4. Design outlook

The current pavilion can be further improved by installing a motor and sensors to transform it into an interactive structure. Instead of manually rotating the turntable, it can be digitally controlled based on current environmental conditions and immediate user-specified commands. Moreover, the pavilion can be sealed using a flexible membrane. In doing so, users can be fully protected against sunlight and wind. By extension, the pavilion can adjust its orientation based on environmental data collected by sensors to increase the level of comfort. Another commonly sought-after feature of interactive architectural forms is the playfulness it creates for urban pedestrians. For this pavilion design, the rotation of the structure can follow human movements. As a person walks through the inner space, the directrix can be dynamically adjusted to center the viewer inside the pavilion. From the viewer's viewpoint, the arches ahead open up, whereas the ones behind close up. The outer directrix can also be designed as a rotatable frame to further increase design possibilities.

### 4. Conclusion

In this paper, the design concept of kinetic elastica-ruled surfaces is used to create novel dynamic architectural forms. Based on the motion behavior, two categories of kinetic elastica-ruled surfaces are developed and discussed. This paper demonstrates that the proposed surfaces can generate continuous motions to achieve stunning visual effects and transform their shapes to different configurations for functional purposes. More significantly, this paper shows that complex 3D dynamic forms can be

accurately, conveniently, and parametrically controlled based on design parameters that control bent states of 2D curves. A full-scale dynamic pavilion was designed and built to demonstrate the effectiveness of the proposed dynamic architectural forms in practical applications. The concept and findings presented in this paper are of significant importance due to the new design possibilities for creating dynamic architectural forms.

### Acknowledgments

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